OSAKA INSTITUTE OF TECHNOLOGY

科研費 KAKENHI

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Diffusion dynamics of QCD matter in nuclear collisions

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Hadron in Nucleus 2025

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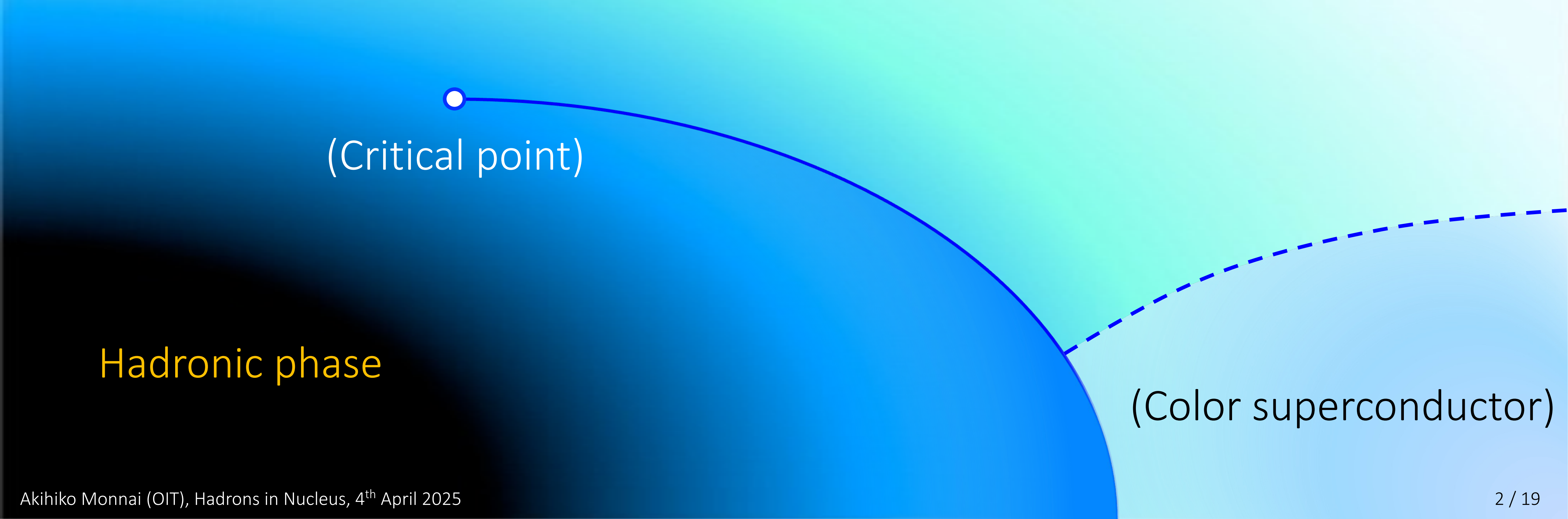
Introduction

- Exploring the QCD Phase diagram

QCD has a rich phase structure depending on the temperature and chemical potentials

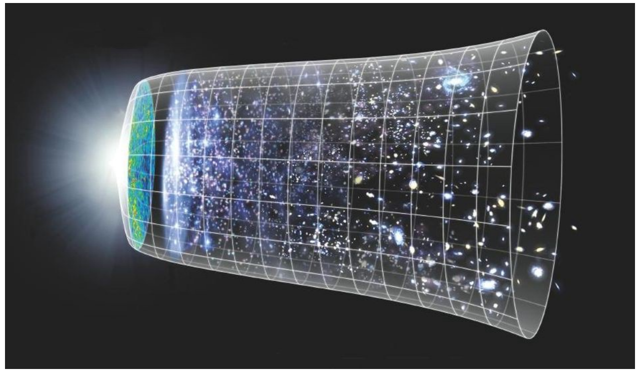


Introduction

- Where is the quark gluon plasma (QGP)?

  - Early universe

    QGP had filled the early universe 10^{-5}-10^{-4}s after the big bang



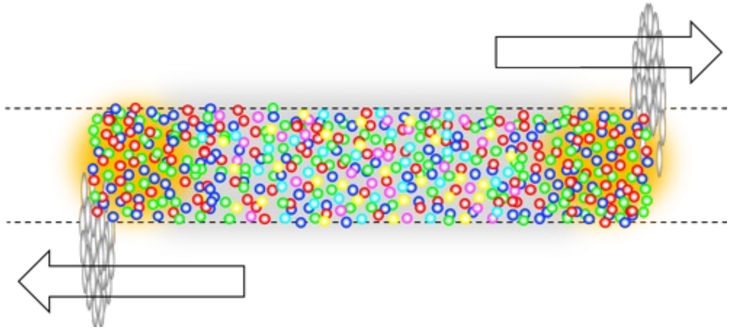
  - Collider experiments ✔

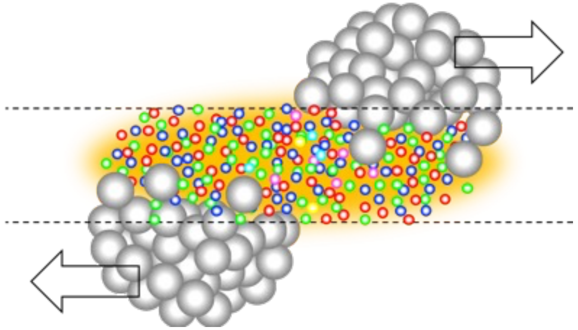
    QGP can be produced by the collision of nuclei (especially heavy ions such as Au, Pb)

    BNL RHIC, CERN LHC (SPS), etc.

Introduction

- Exploring QCD matter at finite density





High-energy nuclear collisions
$\sqrt{s_{NN}} \sim O(10^2 - 10^3)$ GeV

Low-energy nuclear collisions
$\sqrt{s_{NN}} \sim O(1 - 10)$ GeV

Dominated by particles from pair creation

→ High T, low μ_B

Baryon density of nuclei is relevant

→ Low T, high μ_B

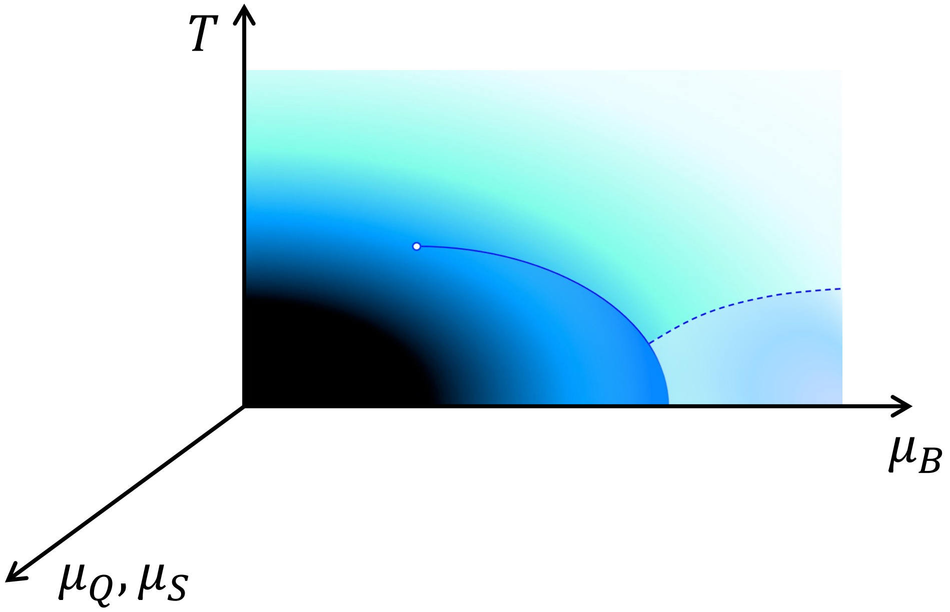
- Performed at RHIC-BES (& SPS), planned at FAIR-CBM, HIAF-CEE, J-PARC-HI, ...

Nuclear collisions

- Conserved charges

The QGP in nuclear collisions are made of light quarks (u, d, s) ($T \sim 200$ MeV)

Baryon (B) | Electric charge (Q) | Strangeness (S) are conserved



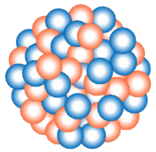
The QCD phase diagram has to be extended to 4 dimensions

- T: Temeperature
- μ_B: Baryon chemical potential
- μ_Q: Charge chemical potential
- μ_S: Strangeness chemical potential

Nuclear collisions

- Geometry

^{197}Au



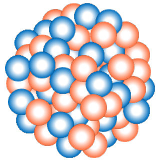
Z = 79

N = 118

$$\frac{Q}{B} \approx 0.401$$

$$S = 0$$

^{208}Pb

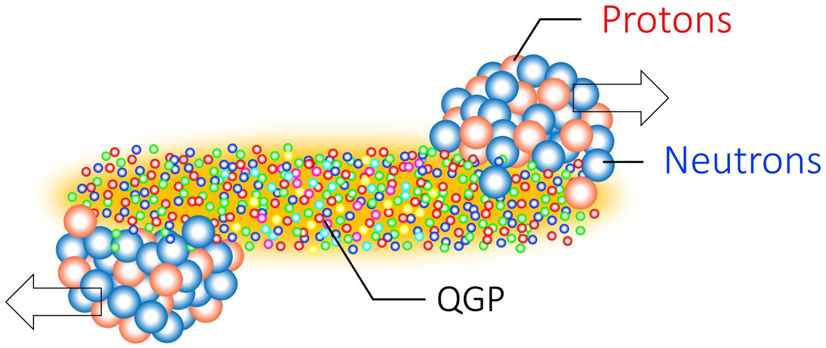


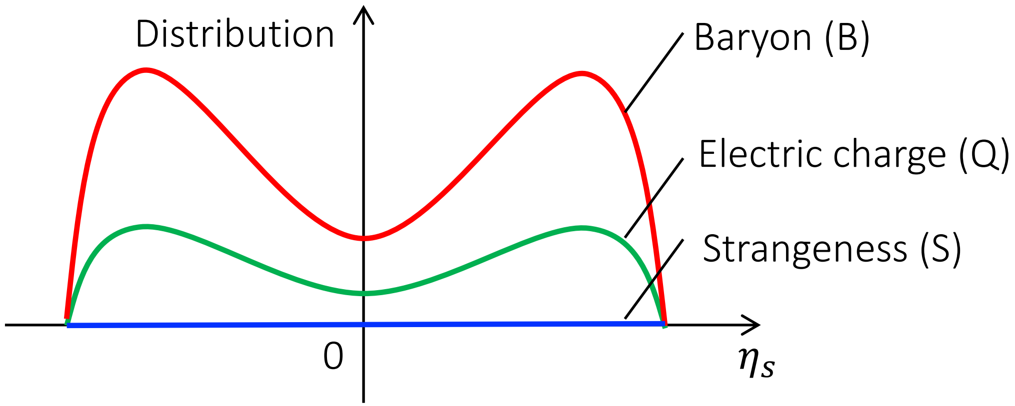
Z = 82

N = 126

$$\frac{Q}{B} \approx 0.394$$

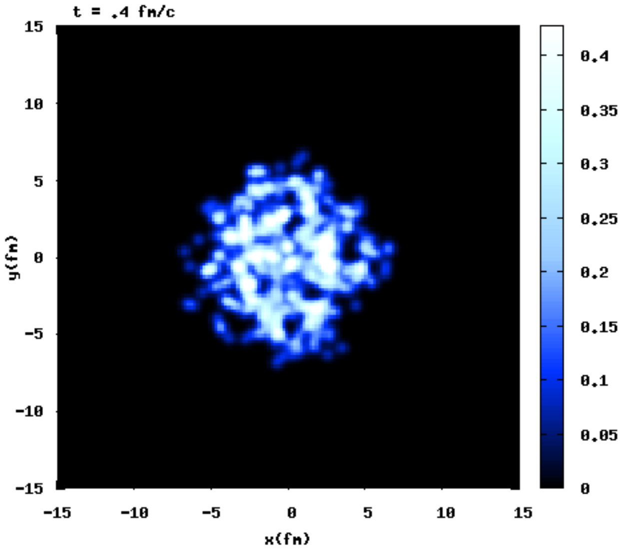
$$S = 0$$



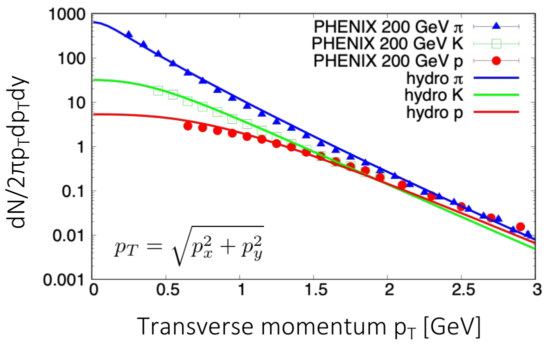


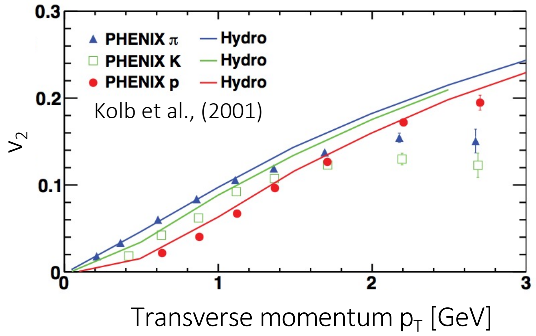
Nuclear collisions

- Fluidity of the QCD matter



Relativistic hydrodynamic model describes nuclear collisions at high energies





We investigate diffusion dynamics of multiple conserved charges (B,Q,S) in the QCD matter

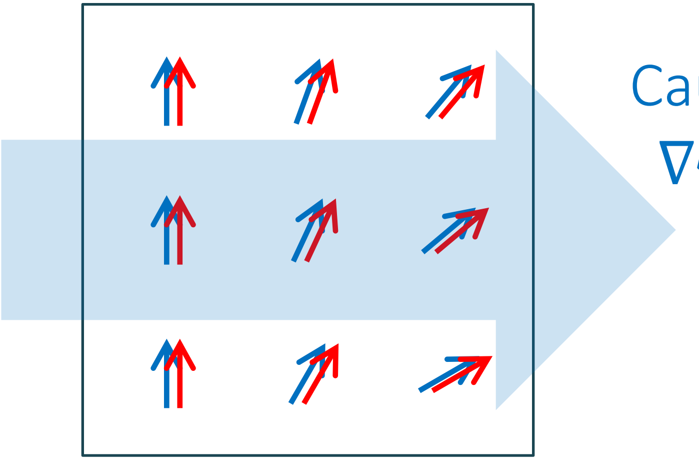
AM, Phys. Rev. C 86, 014908 (2012), Jan A. Fotakis et al., Phys. Rev. D 101, 076007 (2020)

Hydrodynamic model

↑ flow u^μ
↑ conserved current N^μ

- Difference between diffusion and convection

Convection

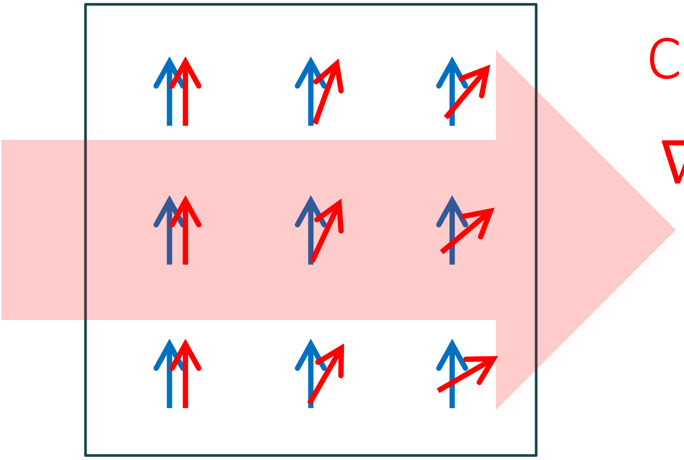


Cause: $\nabla^\mu P$

Charges are carried along with the flow

It can occur in ideal hydrodynamics
(no entropy production)

Diffusion



Cause: $\nabla^\mu \dfrac{\mu}{T}$

Charges are carried against the flow

Only in dissipative hydrodynamics
(finite entropy production)

Hydrodynamic model

- Conserved currents

Energy-momentum tensor	$T^{\mu\nu} = (e+P)u^\mu u^\nu - P g^{\mu\nu}$
Net baryon current	$N_B^\mu = n_B u^\mu + V_B^\mu$
Net charge current	$N_Q^\mu = n_Q u^\mu + V_Q^\mu$
Net strangeness current	$N_S^\mu = n_S u^\mu + V_S^\mu$

*Shear and bulk viscous effects are orthogonal and omitted for simplicity

- Equations of motion

$$\boxed{\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N_B^\mu = 0, \quad \partial_\mu N_Q^\mu = 0, \quad \partial_\mu N_S^\mu = 0}$$

+ Constitutive relations

Hydrodynamic model

- Relativistic diffusion equations

$$\begin{pmatrix} V_B^\mu \\ V_Q^\mu \\ V_S^\mu \end{pmatrix} = \mathcal{K} \nabla^\mu \underbrace{\begin{pmatrix} \mu_B/T \\ \mu_Q/T \\ \mu_S/T \end{pmatrix}}_{\text{Spatial derivatives of fugacities}} - \mathcal{T} \underbrace{D \begin{pmatrix} V_B^\mu \\ V_Q^\mu \\ V_S^\mu \end{pmatrix}}_{\text{Time derivative of diffusion currents}}$$

Conductivity: $\mathcal{K} = \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix}$

Symmetric for Onsager reciprocal relations
Positive semi-definite for the 2nd law of thermodynamics

Relaxation time: $\mathcal{T} = \begin{pmatrix} \tau_{BB} & \tau_{BQ} & \tau_{BS} \\ \tau_{QB} & \tau_{QQ} & \tau_{QS} \\ \tau_{SB} & \tau_{SQ} & \tau_{SS} \end{pmatrix} = \tau_R I$

Assumed to be proportional to identity matrix for simplicity

Numerical setup

- 1+1D hydrodynamic model

Initial conditions: ~ 17.3 GeV Pb+Pb collisions at 0-5% centrality NA49, PRL 82, 2471 (1999)

$$e(\tau_{\mathrm{hyd}},\eta_s) = a_1 \exp(-a_2\eta_s^2 - a_3\eta_s^4)$$
$$n_B(\tau_{\mathrm{hyd}},\eta_s) = n_B^+(\eta_s) + n_B^-(\eta_s)$$
$$n_Q(\tau_{\mathrm{hyd}},\eta_s) = 0.4\, n_B(\tau_{\mathrm{hyd}},\eta_s)$$
$$n_S(\tau_{\mathrm{hyd}},\eta_s) = 0$$

$$n_B^\pm(\eta_s) = \begin{cases} b_1 \exp[-b_2(\eta_s \mp \eta_0)^2 - b_3(\eta_s \mp \eta_0)^4] & (\eta_s > \pm\eta_0) \\ b_1 \exp[-\tilde{b}_2(\eta_s \mp \eta_0)^2 - \tilde{b}_3(\eta_s \mp \eta_0)^4] & (\eta_s \le \pm\eta_0) \end{cases}$$

Hydrodynamization time: $\tau_{\mathrm{hyd}} = 3$ fm/c

Transport coefficients: $\dfrac{\kappa_B}{T^2} = 0.05,\ \dfrac{\kappa_Q}{T^2} = 0.02,\ \dfrac{\kappa_S}{T^2} = 0.1$

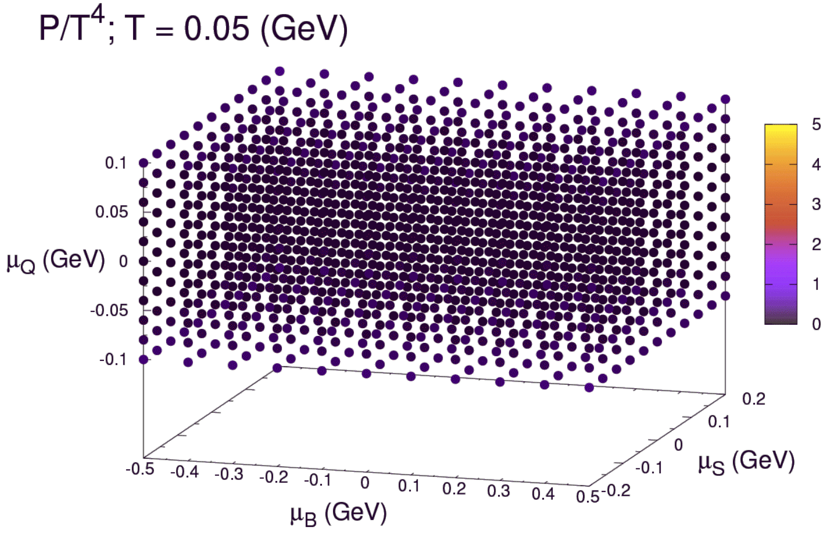
$$\kappa_{BQ} = c\sqrt{\kappa_B\kappa_Q},\ \kappa_{BS} = c\sqrt{\kappa_B\kappa_S},\ \kappa_{QS} = c\sqrt{\kappa_Q\kappa_S} \quad (c = 0 \text{ or } 0.5)$$

Numerical setup

- 1+1D hydrodynamic model

Equation of state: Lattice QCD + hadron resonance gas model (NEOS-4D)

AM, G. Pihan, B. Schenke, C. Shen, Phys. Rev. C 110, 044905 (2024)



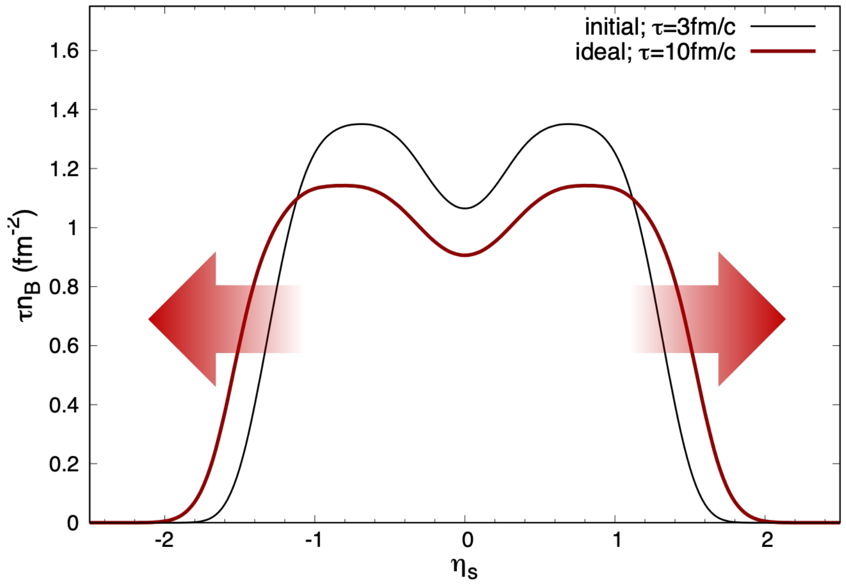
A full 4-dimensional EoS with B, Q, S charges in the range relevant to nuclear collisions

The EoS is available for downloading:

https://sites.google.com/view/qcdneos4d/home

Results

■ Spacetime rapidity distribution of net baryon density (n_B)

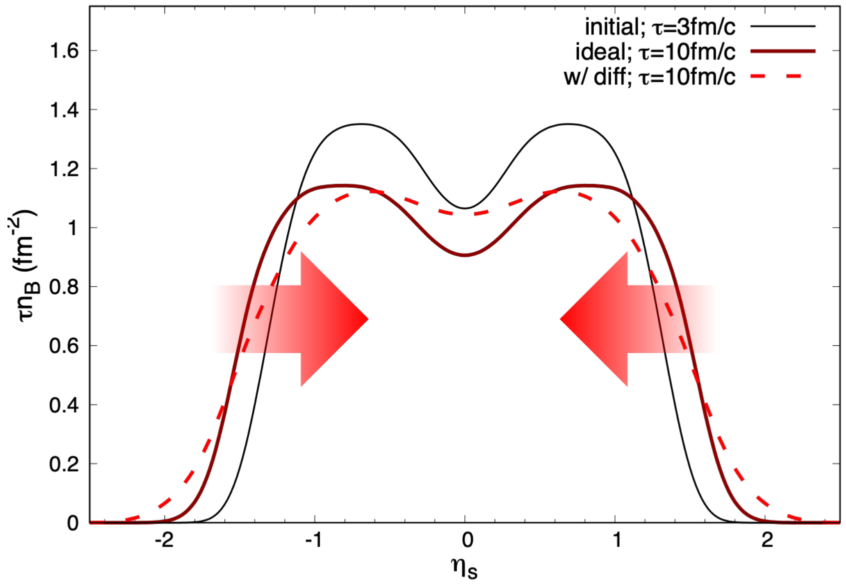


1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

Results

- Spacetime rapidity distribution of net baryon density (n_B)



1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

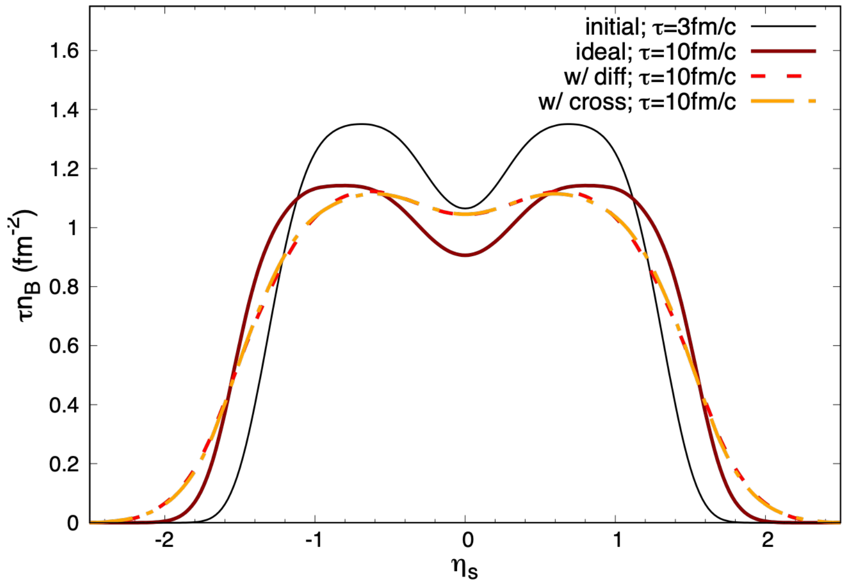
2. With diffusion currents

Partially brought back to mid-rapidity by gradients in μ_B/T

$$V_B^\mu \approx \kappa_{BB}\nabla^\mu \frac{\mu_B}{T} + \kappa_{BQ}\nabla^\mu \frac{\mu_Q}{T} + \kappa_{BS}\nabla^\mu \frac{\mu_S}{T}$$

Results

- Spacetime rapidity distribution of net baryon density (n_B)



$$V_B^\mu \approx \kappa_{BB} \nabla^\mu \frac{\mu_B}{T} + \kappa_{BQ} \nabla^\mu \frac{\mu_Q}{T} + \kappa_{BS} \nabla^\mu \frac{\mu_S}{T}$$

1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

2. With diffusion currents

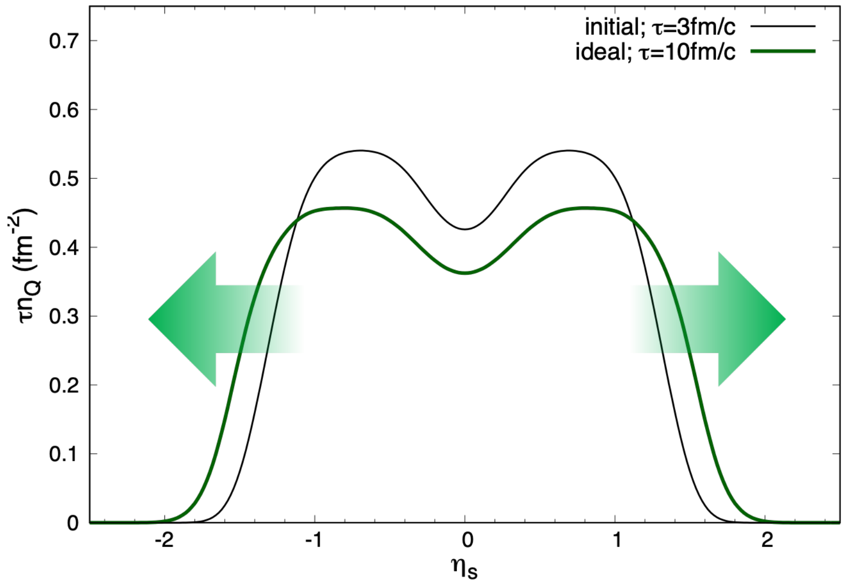
Partially brought back to mid-rapidity by gradients in μ_B/T

3. With cross-coupling currents

Effects of the other charges are negligible

Results

- Spacetime rapidity distribution of net charge density (n_Q)

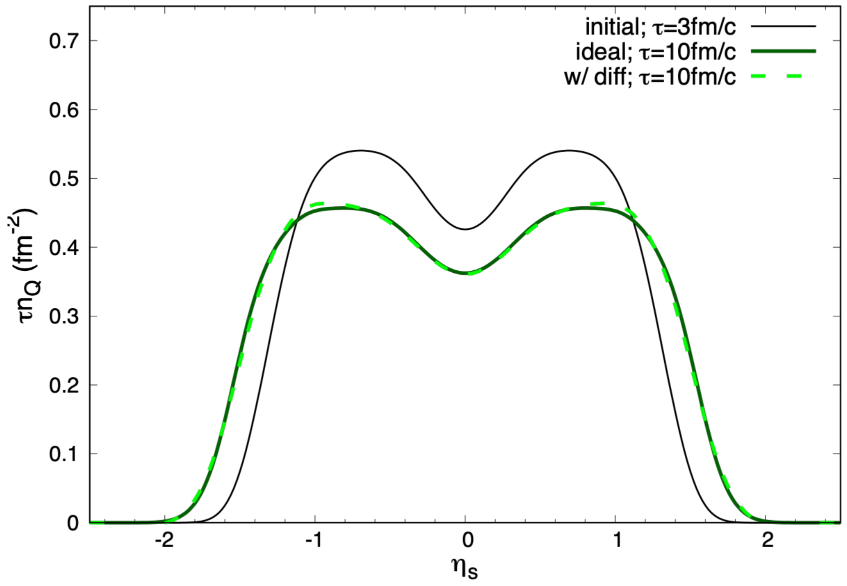


1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

Results

- Spacetime rapidity distribution of net charge density (n_Q)



1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

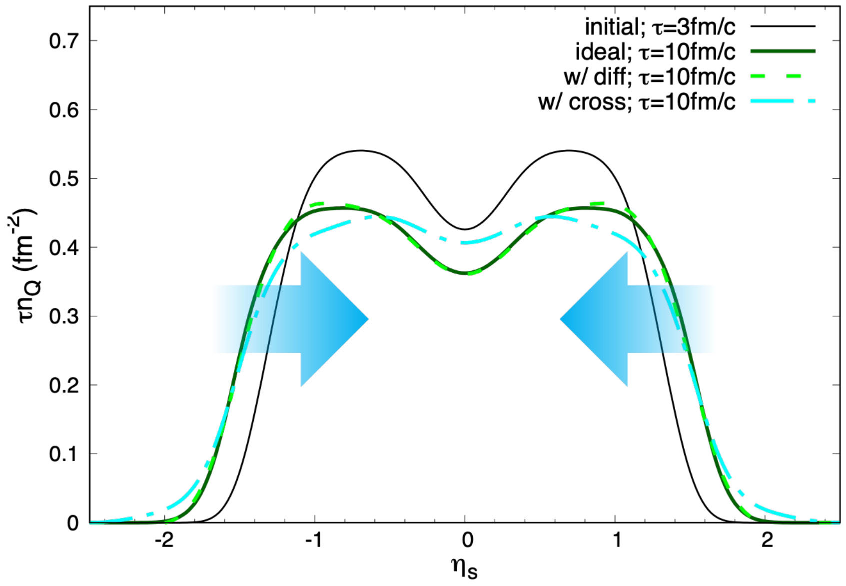
2. With diffusion currents

Carried to forward rapidity, but the effect is small because μ_Q is small

$$V_Q^\mu \approx \kappa_{BQ}\nabla^\mu \frac{\mu_B}{T} + \kappa_{QQ}\nabla^\mu \frac{\mu_Q}{T} + \kappa_{QS}\nabla^\mu \frac{\mu_S}{T}$$

Results

■ Spacetime rapidity distribution of net charge density (n_Q)



$$V_Q^\mu \approx \kappa_{BQ} \nabla^\mu \frac{\mu_B}{T} + \kappa_{QQ} \nabla^\mu \frac{\mu_Q}{T} + \kappa_{QS} \nabla^\mu \frac{\mu_S}{T}$$

1. Ideal hydrodynamic evolution

Carried to forward rapidity by convection

2. With diffusion currents

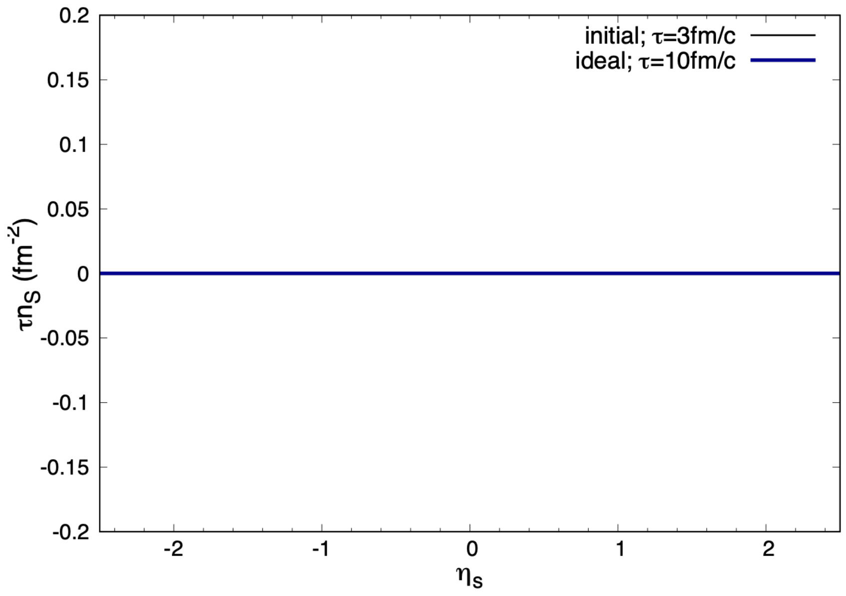
Carried to forward rapidity, but the effect is small because μ_Q is small

3. With cross-coupling currents

Carried back to mid-rapidity by μ_B

Results

- Spacetime rapidity distribution of net strangeness density (n_S)

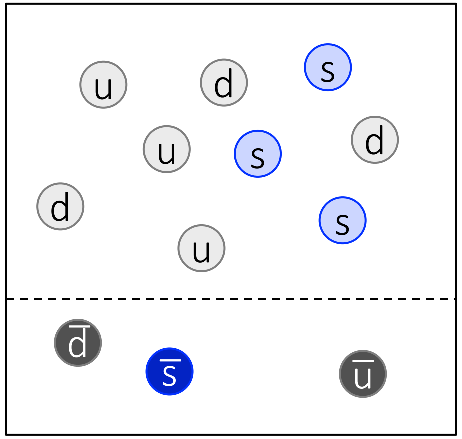


1. Ideal hydrodynamic evolution

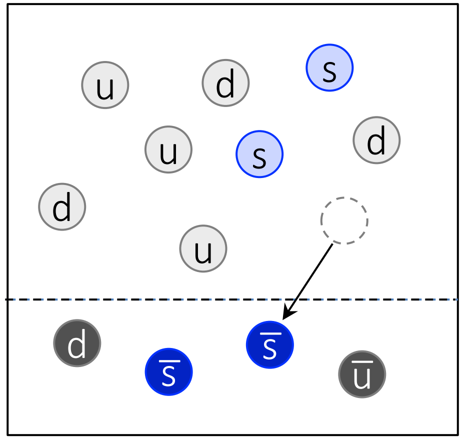
Zero everywhere during time evolution

Discussion

- Strangeness neutrality condition



$\mu_B > 0$ and $\mu_S = 0$
means $n_S \neq 0$

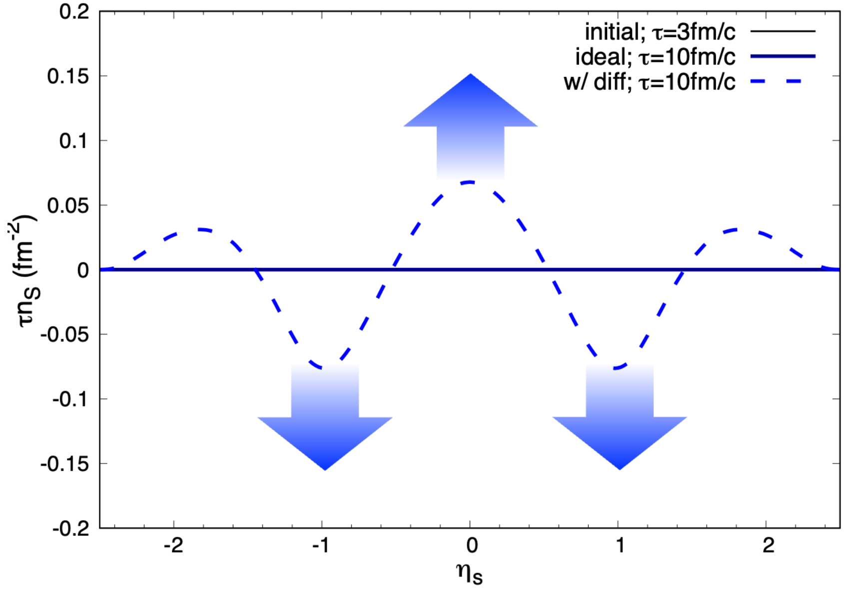


$\mu_B > 0$ and $n_S = 0$
leads to $\mu_S > 0$

s quark chemical potential $\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S = 0$ implies $\boxed{\mu_S > 0}$
when $\mu_B > 0$ and $\mu_Q < 0$

Results

- Spacetime rapidity distribution of net strangeness density (n_S)



1. Ideal hydrodynamic evolution

Zero everywhere during time evolution

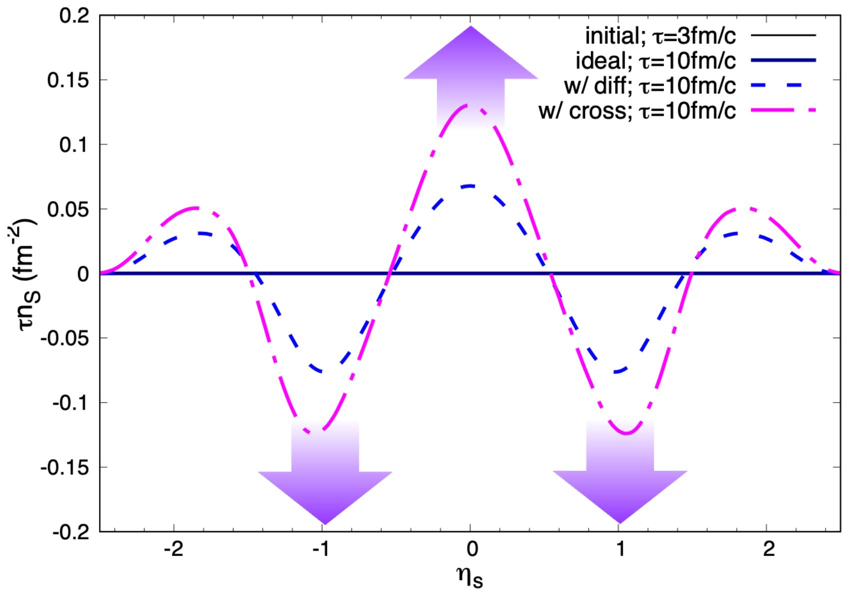
2. With diffusion currents

Gradients in μ_S/T induce non-zero n_S (integrated strangeness is zero)

$$V_S^\mu \approx \kappa_{BS}\nabla^\mu \frac{\mu_B}{T} + \kappa_{QS}\nabla^\mu \frac{\mu_Q}{T} + \kappa_{SS}\nabla^\mu \frac{\mu_S}{T}$$

Results

- Spacetime rapidity distribution of net strangeness density (n_S)



$$V_S^\mu \approx \kappa_{BS} \nabla^\mu \frac{\mu_B}{T} + \kappa_{QS} \nabla^\mu \frac{\mu_Q}{T} + \kappa_{SS} \nabla^\mu \frac{\mu_S}{T}$$

1. Ideal hydrodynamic evolution

Zero everywhere during time evolution

2. With diffusion currents

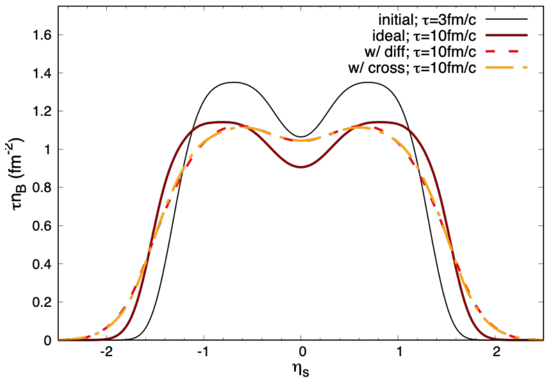
Gradients in μ_S/T induce non-zero n_S
(integrated strangeness is zero)

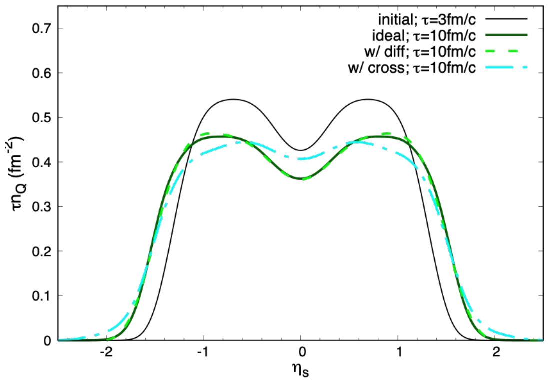
3. With cross-coupling currents

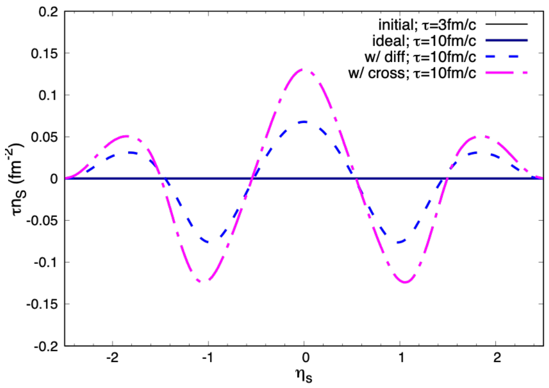
The amplitude is modified by μ_B

Summary and outlook

- Diffusion currents of net baryon, electric charge, and strangeness are introduced in the hydrodynamic model of nuclear collisions
  - Baryon diffusion: n_B is carried inwards; cross-coupling effect is minute
  - Charge diffusion: Small effect; cross-coupling effect is large
  - Strangeness diffusion: Non-zero n_S is induced (the neutrality condition is broken); cross-coupling effect is large

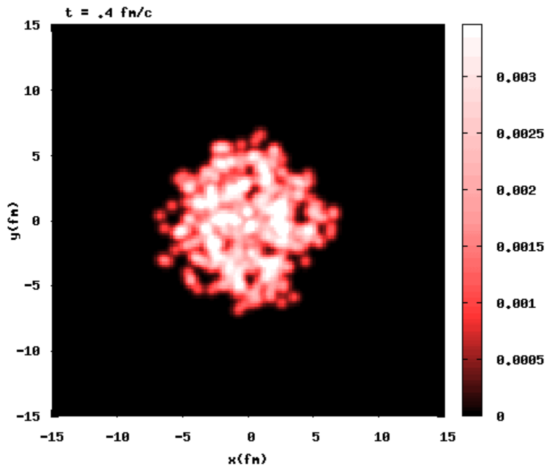


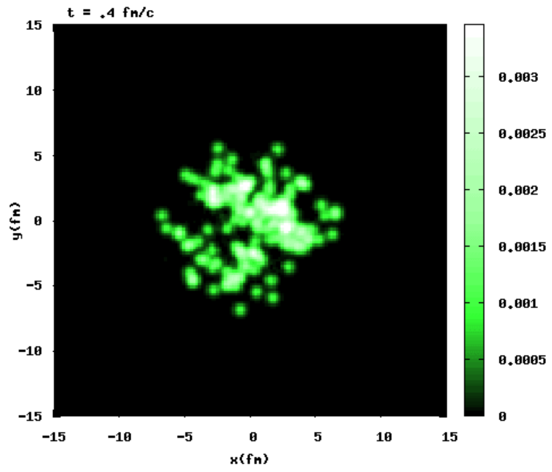


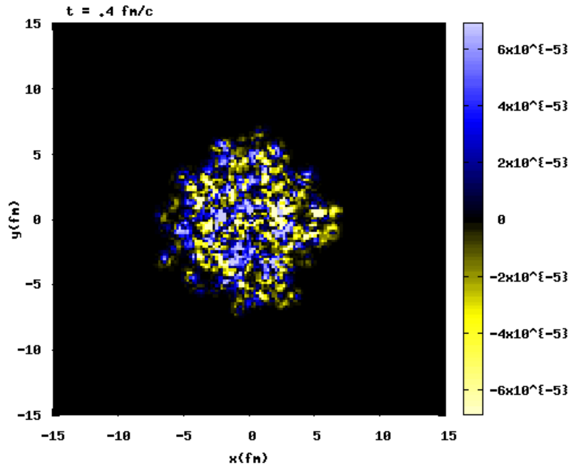


Summary and outlook

- Prospects
  - Full 3D simulations by introducing transverse dynamics to estimate particle spectra and elliptic flow
  - Estimations of initial conditions and transport coefficients, etc.







The end

- Thank you for listening!